COUNTING STATISTICS AFFECTED BY FINITE RESPONSE TIME OF X-RAY DETECTION SYSTEM

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It is usually assumed that the statistical error of diffraction intensity is equal to the square root of observed counts, when the intensity is measured by a counting method. The assumption will be justified, if the event that a detector counts a signal pulse is independent of events that the detector counts other pulses. However, it is obvious that counting of a pulse cannot be independent of counting other pulses, when the effect of counting loss is not negligible. Conventional theoretical models for counting loss [1] predict the mean and variance of observed pulses of rate r counted for a duration of T are given by $m = rT / (1 + r\tau)$ and $\sigma^2 = rT / (1 + r\tau)^3$ for non-extended dead-time τ , and $m = rT \exp(- r\tau)$ and $\sigma^2 = rT \exp(- r\tau)$ $[1 - 2r\tau \exp(- r\tau)]$ for extended dead-time τ.

Detailed experiments based on the Chipman's method [2] have revealed that the counting-loss characteristics of real x-ray detection systems are better modeled by an intermediately-extended dead-time model [3], where the mean observed count is given by $m = r'T \exp(-r'\tau_2)$, $r' = r / (1 + r\tau_1)$, $\tau_1 = \tau - \tau_2$ and $\tau_2 = \rho^{1/2}\, \tau$, for the total dead-time τ and the degree of extension of the dead-time ρ. The formula appears to simulate a serial series of non-extended and extended dead-time models, but it should be assumed that the output pulses of the non-extended model become independent of each other before they are input to the extended dead-time model, for the consistency of the formula. Even though it does not seem to be a realistic assumption, a hypothetical formula of the statistical variance for the intermediately-extended dead-time model can be derived by applying the assumption as $\sigma^2 = r'T \exp(- r'\tau_2)$ $[1 - 2r'\tau_2 \exp(- r'\tau_2)]$.

The statistical variance of x-ray intensities measured with detection systems of laboratory and synchrotron powder diffractometers were experimentally investigated by repeated measurements based on the Chipman's method. It has been found that the experimentally evaluated statistical variance are satisfactorily reproduced by the above model. The model can also be applied to estimate the errors of the intensities corrected for counting loss, which is generally larger than the square root of the measured counts [4].

References

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