QUICK X-RAY REFLECTIVITY OF SPHERICAL SAMPLES

Krassimir Stoev\(^1\), Kenji Sakurai\(^{2,3}\)

\(^1\) AECL – Chalk River Laboratories, Chalk River, Ontario, Canada
\(^2\) University of Tsukuba, 1-1-1 Tennodai, Tsukuba, Ibaraki, Japan
\(^3\) National Institute for Materials Science, 1-2-1 Sengen, Tsukuba, Ibaraki, Japan

ABSTRACT

Recently, a new experimental setup for quick X-ray reflectivity measurements was proposed, which is based on simultaneous recording of an X-ray reflectivity curve over all angles of interest. This new setup for quick X-ray reflectivity allows measurements to be done within seconds, thus permitting studies of the time evolution of chemical, thermal, and mechanical changes at the surfaces and interfaces of different materials. Since the quick X-ray reflectivity measurement setup utilizes an extended X-ray source and a detector, it is important to develop models and to account for the following two effects: (i) diffuse scattering associated with different points of the source and (ii) sample curvature. Models accounting for both effects are presented, and their influences on interpretation of the quick X-ray reflectivity measurement results are discussed.

I. INTRODUCTION

X-ray reflectivity is a non-destructive testing technique used to investigate the structure of surfaces, buried interfaces, thin films, and multi-layers. Classical X-ray reflectivity is a relatively slow technique, with a typical time for one scan on the order of hours. Recently, a new experimental setup for quick X-ray reflectivity (q-XRR) measurements was proposed (Sakurai and Mizusawa, 2007; Sakurai et al., 2007a; Sakurai et al., 2007b; Naudon et al., 1989; Naudon et al., 1992; Chihab and Naudon, 1992; Niggemeier et al., 1997; Koppel, 1997; Albouy and Valerio, 1997), which is based on simultaneous recording of the X-ray reflectivity curve over all angles of interest. The new setup for quick X-ray reflectivity will allow measurements to be done within seconds, thus permitting studies of the time evolution of chemical, thermal, and mechanical changes at the surfaces and interfaces of different materials. Since the quick X-ray reflectivity measurement setup utilizes an extended X-ray source, it will be important to develop models and to account for two effects:

- Both specular reflection and diffuse scattering associated with different points of the source can reach each pixel of the detector.
- Sample curvature (for example, surface of liquid samples) will lead to changes in the recorded X-ray reflectivity curve in comparison to flat samples.

Models accounting for both effects will be presented, and their influence on interpretation of the measurement results from quick X-ray reflectivity will be discussed. For spherical
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samples at glancing incidence, because the source is very close to the sample, one has to consider the strong aberrations in forming the source image. Due to strong aberration effects, the image of the line source will not be a line, but will have a complex 3D shape, i.e., it will be blurred. Generally speaking, there will be two images formed: one by the rays parallel to the incidence plane (tangential image); and one by the rays perpendicular to the incidence plane (sagittal image). Both concave and convex spherical surfaces of the sample will be considered, and the contribution of sample curvature to the measured signal will be evaluated by means of computer simulations. Such simulations are very important to interpretation of the results from quick X-ray reflectivity measurements, because diffuse scattering and sample curvature effects can cause changes in the shape of the X-ray reflectivity curve that are similar to the changes introduced by the sample structural parameters such as density, roughness, layer thickness, etc.

II. MEASUREMENTAL

A schematic of the q-XRR setup is shown in Fig. 1. The real experimental setup used for the modeling done in this paper is shown in the inset in Fig. 1. The linear source has a height of around 10 mm, which at source-to-slit distance ($L_{SK}$) of 270 mm provides incident angles in the range 0 to 37 mrad on the sample. The source emits the Cu $K\alpha$ line (8.047 keV). The slit-to-detector distance is 1800 mm. The slit height ($H_K$) is 30 $\mu$m, and the detector is a linear pixel array (1024 pixels, with detector pixel height $D_H = 50$ $\mu$m). This corresponds to a maximum reflection angle of 28 mrad.

For the purpose of this work, the words “sample” and “mirror” will both be used for describing the reflector. In order to facilitate comparison of the results from different models, one and the same sample was used for all calculations. A multi-layer sample (10 bi-layers of 3-nm Ni / 3-nm C, on a Si substrate) was selected, because it produces X-ray reflectivity scans which have both a Bragg-peak and side-lobes (fringes).
III. MODELING THE EFFECT OF DIFFUSE SCATTERING

Due to the use of an extended source and detector in the q-XRR, each detector pixel will see the specular reflection from the corresponding source point plus the diffuse scattering from all source points, i.e., the diffuse scattering contribution has to be integrated over all incident angles \( \varphi_{\text{in}} \) for each specular angle \( \varphi_{\text{out}} \). Preliminary results on the contribution of the diffuse scattering to the q-XRR scans for a polished surface and a thin film sample were reported by the authors in a previous publication (Stoev et al., 1997). Calculations were done using DS2D software (Mizusawa et al., 2003) and TRDS software (Stepanov, 2012). The results from modeling the multi-layered sample are shown in Figure 2 for different combinations of vertical roughness \( S \), correlation length \( L \), and Hurst parameter \( H \).

As can be seen, the integrated diffuse scattering does not change significantly the shape of the q-XRR curve up to angles of 15 mrad. Above that angle, the amplitude of the side-lobes is reduced, but their position and width are not changed. Unlike classical scanning X-ray reflectivity, the dynamic range of the detectors for q-XRR is not so wide (usually on the order of 4-4.5 decades for 16-bit digitization). If only the region below 15 mrad is used for fitting and extracting of sample parameters from experimental q-XRR scans, there is no need to include the contribution of the diffuse scattering in the modeling function. These results are similar to the ones previously published (Stoev et al., 2009).
IV. MODELING THE EFFECTS OF X-RAY BEAM DIVERGENCE

The sample curvature will generally have the same effect on the reflectivity curve as introducing beam divergence. The effects of X-ray beam divergence for the classical X-ray reflectivity setup (with slits/collimators at both source and detector) were modeled with RefleX6 software (Stoev et al., 1997; Stoev and Sakurai, 2011).

![Effect of Beam Angular Divergence on X-Ray Reflectivity](image)

**Fig. 3. Effect of beam divergence in classical X-ray reflectivity.**

The results are presented in Fig. 3, and can be used for comparison to the results of modeling the q-XRR setup with curved samples. As can be seen, the increase in X-ray beam divergence leads to:

- Reduction in the intensity and increase of the width of the Bragg peak, with practically no change in the position of the Bragg peak.
- Significant change in the slope of the X-ray reflectivity curve around the critical angle of total reflection.
- Significant change in the intensity and shift in the position of the high-frequency fringes (also known as side-lobes). These changes are observed both around the Bragg peak and in the region around the critical angle of total reflection.
V. MODELING SAMPLE CURVATURE EFFECTS FOR CONVEX SAMPLES

For comparison only, the previously developed software RefleX6 (Stoev et al., 1997; Stoev and Sakurai, 2011) was used to model the effects of the radius of curvature for convex samples on the shape of the q-XRR reflectivity curves. As discussed in (Stoev and Sakurai, 2011), there are two preferred ways to perform such modeling. The first approach is to use a 2D numerical integration of the X-ray reflectivity over the whole sample for each detector point. The sample is divided into $N_S$ sub-intervals and, for each detector pixel and each sample sub-interval, the corresponding source point and reflection angle are calculated. Corrections are made for the shadowing effects due to the knife-slit, self-shadowing due to the mirror curvature, and for the finite sample size. This model correctly accounts for the changing reflection angle from sub-interval to sub-interval and for the detector acceptance angle for each sub-interval. Results from this model are shown in Fig. 4.

![Fig. 4. Effect of sample curvature on q-XRR scans (numerical integration).](image)

Although very simple and easy to use, the model described above has some disadvantages, and more specifically it assumes a cylindrical sample, not a spherical sample. In order to overcome this deficiency, another model was proposed (Stoev and Sakurai, 2011), which accounts for the 3D shape of the spherical mirror. This is done by using the source image rather than the source itself. For spherical samples at glancing incidence, because the source is very close to the sample, there are very strong...
aberration effects in forming the source image. Discussion on this subject can be found in Bridou, 1994. Two images will be formed: a tangential image (formed by the rays parallel to the incidence plane); and a sagittal image (formed by the rays perpendicular to the incidence plane). This will lead to blurring of the image, i.e. the image of a line-source will not be a line, but will have a complex 3D shape. At glancing incidence, the position of the sagittal image does not change significantly with incident angle. On the other hand, the position of the tangential image depends very strongly on the incident angle. The calculation of the reflectivity from a 3D spherical mirror is again based on numerical integration. The source is divided in small sub-sections, the sample is also divided in $N_S$ sub-intervals, and for each source sub-section and sample sub-section, the reflection angle, the tangential image, and the corresponding detector pixel are determined, and the calculated intensity is added to the total intensity for the corresponding detector pixel. For each source sub-section and sample sub-section, the position of the tangential image is used to calculate the correction factor (i.e., correction for the different angular density of the beam energy due to the different divergence for the beams reflected from a flat and convex sub-interval). Results from calculations based on the 3D model are shown in Fig. 5. There is no significant difference between the two models for convex samples (see Figs. 4 and 5), mainly because convex mirrors always produce beams that have larger divergence than the beams produced by flat mirrors, i.e. there will be a slight reduction of the angular density of the beam energy for convex mirrors when compared to flat mirrors.

Fig. 5. Effect of sample curvature on q-XRR scans (3D model).
VI. MODELING SAMPLE CURVATURE EFFECTS FOR CONCAVE SAMPLES

The modeling setup for the q-XRR scans from a concave mirror is shown in Fig. 6. In this case, several additional shadowing effects have to be considered:

- For concave mirrors, it is possible to have sections of the line-source or of the detector that are completely shadowed, i.e. from which no point of the mirror surface is visible. The highest point for these sections is defined as the intersection point of the source (or the detector) with the line through knife-end and mirror-end. For example, in Fig. 6, all points in the brown sections of the source will not “see” any point of the top-surface of the mirror.

- Self-shadowing of the mirror will occur when some parts of the mirror are blocking other parts of the mirror from seeing the rays from the source (or to the detector). For example in Fig. 6, all source points below \( h(Q) \) are blocked by the left end of the mirror from seeing the parts of the mirror to the left of point \( Q \).

![Fig. 6. q-XRR setup for a concave mirror.](image)

Again, two approaches were considered: 2D numerical integration over a cylindrical concave mirror, and 3D numerical integration while accounting for changes in the position of the tangential image. Both methods were implemented in a revised version of the RefleX6 software.

The 2D numerical integration is based on dividing the sample into \( N_S \) sub-intervals and, for each detector pixel and each sample sub-interval, calculating the corresponding source point and reflection angle. Corrections are made for all shadowing effects and for the finite sample size. Results from this model are shown in Fig. 7. This model correctly accounts for the changing reflection angle from sub-interval to sub-interval, and for the acceptance angle for each sub-interval. The main problem with this model is that it...
assumes that each mirror sub-section is a flat mirror, i.e. it does not account for the additional beam divergence (or convergence) introduced by the curvature of the mirror sub-section.

![Fig. 7. Integrated 2D q-XRR reflectivity for a concave mirror.](image)

For a concave mirror, we can have discontinuities in the tangential image. This was partially discussed in Bridou, 1994. The sagittal and the tangential image points are introduced for convenience only, and do not represent the true “blurred” image as formed by complete ray-tracing of the optical system. The position of the sagittal and tangential image points can be calculated using standard formulae (also known as Coddington Equations):

\[
\frac{1}{d_0} + \frac{1}{d_s} = \frac{2.\sin(\theta)}{R} \\
\frac{1}{d_0} + \frac{1}{d_T} = \frac{2}{R.\sin(\theta)}
\]

The sign convention is as follows: \(d_0, d_s, d_T,\) and \(R\) are positive to the left of the vertex point, and negative to the right of the vertex point, while \(\theta\) is always positive. For small angles \(\theta\), the sagittal image position \(d_s\) does not depend significantly on \(\theta\), and its value
is close to $d_0$, while tangential image position $d_T$ depends strongly on $\theta$. There will be an angle $\theta$, for which $d_T = \infty$, i.e. we will have a discontinuity in the tangential image. This corresponds to having parallel beams, for which no image is formed. Again, these formulae are only an approximation, because they do not consider that many parts of the mirror are involved in the creation of the image.

An illustration of this problem is shown in Fig. 8, for the case of a concave mirror with a radius of curvature of 30 m. This mirror transforms the red part of the source into the purple part of the tangential image (image is “above” the mirror), while the green part of the source is transformed by the mirror into the yellow part of the tangential image (i.e., image is “below” the mirror). The source point between the red and the green part of the source does not produce an image, it produces a parallel beam. The tangential image for the case described above was calculated based on the rays passing through the ends of the mirror.

Another problem is that different parts of the mirror form beams with different divergence, i.e. different tangential images. Also, there are several effects which contribute to the energy density of the beam on the detector pixels for the case of flat and convex mirrors:

- Different parts of the mirror will contribute to each detector pixel.
- Different mirror lengths will correspond to the pass-through beam for each source point.

Fig. 8. Discontinuities in the tangential image.
A different number of detector pixels will be illuminated by the pass-through beam for each source point. All these effects have to be considered simultaneously when modeling q-XRR scans of concave mirrors. This is done by evaluating the “illuminated” parts of the detector for each source point and each mirror subsection, for both flat (\(H_F\)) and concave (\(H_C\)) mirrors, as illustrated in Fig. 9. The correction factor (\(H_F / H_C\)) is used to normalize final results. In addition to this correction, the results have to be normalized in such a way as to ensure that integration of a flat mirror, by following the procedure for integration and correction of a concave mirror, will produce the same results as the theoretical reflectivity of a limited flat sample.

![Fig. 9. “Illuminated” parts of the detector for flat and concave mirrors.](image)

It should be noticed that, for specific cases, \(H_C\) could become very close to zero (i.e., the mirror sub-section generates a tangential image onto the detector line, as shown in Fig. 8). In these cases, the correction factor becomes very large. To avoid such cases, the maximum value of the correction factor was restricted based on the sizes of the source pixels and detector pixels, and the “nominal” beam spread from a flat sub-section. Even with this restriction, for some values of radius of curvature \(R\) and nominal reflection angle \(\theta\), there will be large uncertainties in the corrected results, as shown in Fig. 10.

Results from the calculation of the q-XRR scans from concave mirrors with different radii of curvature as per the above described procedure are shown in Fig. 10. The 3D q-XRR scans are similar to the 2D integrated q-XRR scans presented in Fig. 7, but the difference is bigger for the case of concave mirrors than that for convex mirrors (see Figs. 4 and 5). This can be attributed to the larger beam divergence from each section for the concave mirrors in comparison to the convex mirrors, i.e. larger changes and discontinuities in the position of the tangential image for the case of a concave mirror.
Generally speaking, the effects of the sample curvature on the shape of the q-XRR scans for concave mirrors are similar to those for convex mirrors. Usually, these effects are significant when the radius curvature, $R$, is $< 10$ m, and are very similar to the effects of X-ray beam angular divergence in classical X-ray reflectivity (see Figure 3):

- Reduction in the intensity and increase in the width of the Bragg peak, accompanied with small change in the position of the Bragg peak.
- Significant change in the shape and the slope of the q-XRR curve around the critical angle of total reflection.
- Significant change in the intensity and shift in the position of the high-frequency fringes (or side-lobes). These changes are observed both around the Bragg peak and around the critical angle of total reflection.

**Fig. 10. Calculated 3D q-XRR reflectivity for concave mirror.**

**VII. CONCLUSION**

The recently developed quick X-ray reflectivity technique offers unique capabilities for studying fast processes at the surface and interface of nano-scale materials, like the time evolution of chemical, thermal, and mechanical changes at the surface and the interface of different objects or materials. The current study was directed at determining the contribution of diffuse scattering and sample curvature to the measured signal from a quick X-ray reflectivity setup. In some cases, the integrated diffuse scattering can significantly influence the shape of the recorded reflectivity curves with the quick X-ray
reflectivity setup. The effects of the sample curvature on the shape of the reflectivity curve measured with the q-XRR setup are even more severe, manifested as changes in the position and width of the Bragg peak and/or side-lobes. In order to obtain reliable data from the q-XRR measurements, the existing models for description of X-ray reflectivity curves must be modified to include the contribution of the diffuse scattering and to account for sample curvature.

REFERENCES