Monte Carlo Simulation of Pulse Pile Up

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Abstract

The Monte Carlo simulation code CEARPPU has been developed to simulate pulse piled-up spectra for high counting rates from given true spectra. In this simulation, the exponential pulse interval distribution is used and the multichannel analyzer (MCA) is assumed to operate perfectly in obtaining and scoring the first local maximum from a sum pulse. Forcing and other variance reduction techniques are used to accelerate the simulation process. An accurate simulated spectrum can be obtained in two or three minutes with this code when it is run on a Sun Ultra I work station. Simulation results showed excellent agreement with benchmark measurements with an Fe-55 source and a Si(Li) detector. Use of this code in conjunction with a Monte Carlo code that simulates true spectral responses will allow one to simulate the actual pulse height spectra obtained for radiation analyzer systems. This, in turn, will allow the use of a suitable nonlinear approach for the routine analysis of in vivo medical or on-line process samples at very high counting rates.

1. Introduction

There are a number of reasons for using the highest possible radiation detection counting rates in radiation measurement applications. In medical applications, such as the in vivo measurement of lead in bone, this insures that sample movement has minimal effect[1-3]. In industrial on-line continuous measurement applications, one desires maximum precision in minimum measurement time for fast moving and rapidly changing process streams.

The highest possible radiation detection counting rates are presently controlled by the pulse pile-up characteristics of the detection system of interest. In figure 1, Fe-55 source spectra measured with a Si(Li) detector system at various counting rates are presented to illustrate pulse pile-up effects. It is seen that at the highest counting rate a large amount of spectral distortion is observed - both within the range of the true spectrum and above it. While it is possible to reduce pulse pile up with hardware based pulse pile-up rejections, one can never completely eliminate it by this approach. In addition, the spectrum distortion caused by pulse pile-up can be increased by the use of hardware based pulse pile-up rejection. Therefore, it is highly desirable to use software based pulse pile-up models which can, in principle, correct for any amount of pulse pile up. The present paper describes our pulse pile-up modeling based on Monte Carlo simulation.
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Figure 1. Fe-55 Source Spectra measured at various counting rates with Si(Li) Detector

Figure 2. Amplifier Pulse Shape and Sum Pulse from Pile-Up
Our previous work[4-6] on pulse pile-up modeling consisted of an exact deterministic model for double pulse pile up. Every possible double pulse pile-up event was considered which resulted in \( n^2 \) calculations for a true spectrum of \( n \) channels. In principle, this approach could be extended to triple and more piled-up pulses, but that would involve \( n^m \) calculations where \( m \) is the number of piled-up pulses considered. Such calculations would prove to be impractical. At that time we did not consider Monte Carlo simulation since it was thought to be too time consuming compared to the deterministic approach. However, it recently occurred to us that this may not be true since Monte Carlo simulation automatically treats the most probable events, the forcing variance reduction technique can be used, and present computers are much faster.

Bristow and Harrison[7] made an extensive study of pulse pile up - including the use of Monte Carlo simulation. However, their Monte Carlo simulation was primarily for benchmarking other results and not a practical simulation tool. They gave very little information on their Monte Carlo simulation approach. Other research groups have tried pulse pile-up corrections for specific cases[8,9]. Unlike their approach, we are attempting to devise a complete Monte Carlo simulation package by adding a fast pulse pile-up simulator to generate spectral results which can be directly compared with actual measurements.

To accelerate the Monte Carlo simulation, the forcing technique and some other variance reduction schemes were used. In the pulse pile-up simulation, the detector, pre-amplifier, and amplifier are all assumed to work perfectly. The output signal for a pulse train that is a series of pulses which are piled-up, is the simple linear sum of all of the individual pulses. The multichannel analyzer (MCA) is assumed to find only the first local peak in a pulse train (see figure 2). The time interval between two pulses is assumed to be exponentially distributed with a given time constant (the interval distribution).

Simulation results show good agreement with the actual measurements with a Si(Li) detector and an Fe-55 source. The computation time for \( 10^7 \) histories took only 2 or 3 minutes on a SUN Ultra I work station. So it looks like the inversion (which is essentially finding a true spectrum from a measured one) can be practically achieved with this approach by using an appropriate iterative method.

With the help of this pulse pile-up model and a detector response function, a spectrum calculated by a radiation transport/simulation code like MCNP or CEARXRF[10-12] can be converted to a simulated pulse-height spectrum which has a convolved Gaussian spread and pulse pile up. It can then be compared directly with an actual measured spectrum to give a reduced chi-square value that can be used directly in a suitable iterative analysis method.

2. Simulation Modeling

2.1 General Considerations

The measurement system which is to be simulated usually consists of a semiconductor detector, a pre-amplifier, an amplifier and an MCA. Liquid nitrogen cooled Si(Li) and Ge detectors are common choices in X-Ray Fluorescence (XRF) and Prompt Gamma Neutron Activation Analysis (PGNAA) applications because of their excellent energy resolution and reasonable detection efficiency. Usually a pre amplifier is attached to the detector and the
amplifier produces semi-Gaussian shaped pulses to be processed and digitized by an analog-to-digital converter (ADC) which is a major functioning part of an MCA. Only the first local peak in a sequence of pulses which are piled up is detected by the MCA as the resulting pulse height of the pulse train. The rest of the pulse train is ignored until the baseline voltage is restored. It is assumed that all the components in the measurement system work perfectly, independent of experimental counting rate changes.

2.2 Analog Mode

The Monte Carlo simulation approach is a “forward” one that is based on assuming that a true counting rate and spectrum are available. Let \( N_t \) be the true total number of counts in a given counting time \( t_m \) and \( h(E) \) be the true probability distribution function (pdf) of pulse-height energy \( E \). To simulate the pulse piled-up total counts in time \( t_m \) and pulse-height distribution one simply samples \( N_t \) pulse events by first choosing a leading pulse size from the \( h(E) \), then choosing an interval between this pulse and the next from the interval distribution, then choosing the first piled-up pulse from the \( h(E) \) pdf, etc., until the interval between pulses is larger than the pulse width \( \tau \) of a sample pulse. When this occurs the pulse train being generated is terminated. The first local peak in each pulse train is numerically determined and scored in the pulse piled-up spectrum.

The interval distribution required is given by:

\[
f(\Delta t) = \lambda \exp(-\lambda \Delta t)
\]

where \( \Delta t \) is time interval and \( \lambda \) is the true counting rate \( N_t/t_m \). Time intervals are chosen randomly by using the cumulative distribution function (cdf) obtained by integrating the pdf from 0 to \( \Delta t \). This cdf is:

\[
F(\Delta t) = \int_{0}^{\Delta t} f(x)dx = 1 - \exp(-\lambda \Delta t).
\]

It is set equal to the cdf of the uniform random number to obtain \( \Delta t \) from the uniform random number \( R \).

\[
F(\Delta t) = R
\]

\[
\Delta t = -(\frac{1}{\lambda}) \ln(1 - R)
\]

As the input true spectrum is essentially an unnormalized pdf of pulse-height energy in tabular form, the pdf \( h(E) \) and cdf \( H(E) \) can be easily constructed from it. A random pulse-height energy is chosen by setting this cdf equal to the cdf of the uniform random number \( R \) and solving for \( E \).
\[
H(E) = \int_{x=0}^{E} h(x) \, dx = R
\]

\[
E = H^{-1}(R)
\]

Where \( H^{-1} \) is the inversion function of \( H \).

In simulation, the actual shape of the amplifier pulses should be used for the best result. Figure 2 shows the pulse shape used in the present case which comes from the ORTEC amplifier model 572[13] and the pulse trains formed by double pulse pile up. Usually only the observed counting rate, \( m \), is known. In this case the true counting rate \( \lambda \) associated with the measurement can be obtained by numerical inversion from the paralyzable model[14] for a given pulse width \( \tau \) which is assumed to be constant and independent of the pulse height.

\[
m = \lambda \, \exp(-\lambda \tau)
\]

2.3 Forcing Mode

The analog mode wastes computation time by sampling single pulse trains whose spectral output is exactly the same as the input true spectrum. To avoid this and accelerate the simulation, one can simulate only pulse trains with two or more pulses piled up by forcing every second pulse to be piled up on the previous pulse (forcing mode). Then the already known spectral response from single pulse trains is added.

When \( N_t \) is the total number of true pulses, the number of true pulses occurring in the single pulse trains \( N_{st} \) and the number of true pulses occurring in the pulse trains with two or more pulses piled up \( N_{pt} \) can be calculated from the continuity equation, the paralyzable model[14] for \( N_{st} \), and use of the continuity equation with \( N_{st} \) for \( N_{pt} \).

\[
N_t = N_{st} + N_{pt}
\]

\[
N_{st} = N_t \, \exp(-2\lambda \tau)
\]

\[
N_{pt} = N_t [1 - \exp(-2\lambda \tau)]
\]

The observed spectrum or the output spectrum from Monte Carlo simulation \( R_o(E) \) is essentially the sum of the single pulse train response \( R_s(E) \) and the multiple pulse train response \( R_p(E) \).

\[
R_o(E) = R_s(E) + R_p(E) = N_{st} \cdot h(E) + R_p(E)
\]

As the single pulse train response is already known, only the multiple pulse train response from \( N_{pt} \) true pulse simulations which are piled up is calculated in forcing mode.

When one forces a pulse to be piled up on the previous pulse, a modified pdf \( g(\Delta t) \) and
cdf \( G(\Delta t) \) are used as \( \Delta t \) should remain less than the pulse width \( \tau \).

\[
g(\Delta t) = \frac{f(\Delta t)}{F(\tau)} = \frac{\lambda \exp(-\lambda \Delta t)}{1 - \exp(-\lambda \tau)}
\]

\[
G(\Delta t) = \frac{1 - \exp(-\lambda \Delta t)}{1 - \exp(-\lambda \tau)}
\]

\[
\Delta t = G^{-1}(R) = -\lambda / \ln[1 - R \cdot F(\tau)]
\]

2.4 Simulation of Hardware Pulse Pile Up Rejection

Along with a choice of analog or forcing mode, one can simulate hardware based pulse pile-up rejection by adding a simple rejection criterion. When the rejection option is chosen, responses from pulse trains whose width is larger than a rejection criterion \( \tau_{\text{rel}} \) are removed from the output spectrum.

3. Simulation Results

Simulation results are compared in Figures 3 and 4 with measured spectra which were taken with a Si(Li) detector and an Fe-55 source. Amplifier shaping times of 2 and 10 \( \mu \)sec were used. Both cases show excellent agreement with the measurements and it is seen that simulation results are accurate and correct at least through the triple pulse pile-up region. The difference in the quadruple pulse pile-up region in figure 3 is thought to come from background counts. Also it is noticed that when the same number of pulses are simulated as are present in the experiment the same degree of random fluctuations as in the measurement can be obtained from simulation.

From Monte Carlo simulation one can extract important information by modifying the bookkeeping routines. Figure 5 shows various components of the pulse piled-up response for the case in figure 4. Spectral information is gathered and grouped according to the number of pulses in a pulse train to give the components. Spectral response from single pulse trains has exactly the same shape as the input true spectrum assumed in section 2. All the higher components also have lower pulse pile-up contributions in them. The shapes of these components look reasonable and might be used in inversion calculations.

Hardware based pulse pile-up rejection is simulated and suggested in figure 6 with similar measurement without rejection. The rejection factor \( \tau_{\text{rel}} / \tau \) used is 1.05. The general shape looks just as expected. Actual comparison with measurement will be presented in a future paper.
Figure 3. Pulse Pile-Up Simulation for Amplifier Shaping Time of 10 μsec

Figure 4. Pulse Pile-Up Simulation for Amplifier Shaping Time of 2 μsec
Figure 5. Spectral Components in Pulse Pile-Up Response (2 µsec Amplifier Shaping Time)

Figure 6. Hardware Based Pulse Pile-Up Rejection Simulation
4. Discussion and Conclusions

A Monte Carlo code has been devised and tested for simulating pulse pile up in radiation detection spectral analysis systems. By using a technique in which every pulse train is forced to contain at least two piled-up pulses, the simulation for any number of piled-up events is obtained very efficiently. A Monte Carlo simulation with excellent precision for a piled-up pulse-height spectrum can be obtained in two or three minutes on a Sun Ultra I Workstation. Benchmark results with an Fe-55 source and a Si(Li) detector and associated electronics indicate that the code is capable of excellent accuracy.

Use of this code with a suitable Monte Carlo radiation transport code and a detector response function allows the simulation of actual observed pulse-height spectra for analyzer systems taken at high counting rates with a large amount of spectral distortion. This will enable one to perform elemental analyses by an iterative technique using the actual uncorrected experimental spectra directly. Use of such high counting rate data will minimize in vivo measurement errors due to body movement and allow the highest possible accuracy for on-line process measurements.

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References

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