INVESTIGATION OF ELECTRON EXCITED X-RAY SPECTRA IN DEPENDENCE ON THE ANGLE OF ELECTRON INCIDENCE

Horst Ebel, Robert Svagera, Johann Wernisch, Maria F. Ebel and Michael Sander

Institut für Angewandte und Technische Physik, Technische Universität Wien
Wiedner Hauptstraße 8 - 10, A 1040 Wien (Austria)

ABSTRACT

From the results of our investigations it can be concluded that the theoretical description of x-ray tube spectra by Love and Scott’s depth distribution of x-ray excitation holds for incidence angles of electrons from $\varphi = 90^\circ$ to $\varphi = 30^\circ$ and for take-off angles of x-rays from $\varepsilon = 90^\circ$ to $\varepsilon = 15^\circ$. All angles are measured with regard to the surface. For these angular ranges the values of $\text{const}$ and $x$ for white x-radiation can be given by

$$\text{const} = 1.077 \times 10^9 \text{ [photons/\text{s}/\text{sr}/\text{mA/keV}]} \quad \text{and} \quad \sigma_{\text{const}} = 13.8\%$$

$$x = 1.111 \quad \text{and} \quad \sigma = 8.8\%$$

and $\text{Const}$ for characteristic radiation becomes

$$\text{Const} = 5.166 \times 10^{13} \text{ [photons/\text{s}/\text{sr}/\text{mA}]} \quad \text{and} \quad \sigma_{\text{Const}} = 9.4\%$$

INTRODUCTION

Electron excited characteristic x-ray spectra can be described by the standard approaches for quantitative electron probe microanalysis. In these approaches different depth distribution functions (DDF) of characteristic x-ray excitation are employed. Green and Cosslett\textsuperscript{1,2} and Heinrich\textsuperscript{3} have shown that the excitation of continuous x-radiation can be treated in analogy to characteristic x-radiation. Wiederschwinger\textsuperscript{4}, Ebel et al\textsuperscript{5}, Schoßmann et al\textsuperscript{6} and Ebel\textsuperscript{7} obtained best agreement between computed and measured continuous spectra applying Love and Scott’s equidistribution model\textsuperscript{8,9,10,11} where the electron range is given by $2 \cdot \varphi$. Love and Scott developed their DDF assuming an electron incidence normal to the surface ($\varphi = 90^\circ$).

Fig. 1 Geometry of electron incidence and x-ray take-off.
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The present investigation deals with a recognition of the incidence angle $\phi$ by a modified electron range $2\rho x \cdot \sin \phi$ in Love and Scott's DDF and experiments performed under $\phi = 90^\circ$ to $\phi = 20^\circ$ and $\varepsilon = 90^\circ$ to $\varepsilon = 5^\circ$ in order to quantify the angular range of validity of this modification for three target materials (Al, Cu and Mo) and three acceleration voltages (10, 20 and 30 kV). The geometry can be seen from Fig. 1.

**THEORY**

In Love and Scott's concept the DDF is assumed to be constant from surface to depth $2\rho x$. 

$$\dot{\rho} x = \rho x_m \cdot \frac{0.49269 - 1.0987 \cdot \eta + 0.78557 \cdot \eta^2}{0.70256 - 1.09865 \cdot \eta + 1.0046 \cdot \eta^2 + \ln U_0} \cdot \ln U_0$$

$$\rho x_m = \frac{A}{Z} \left( 0.787 \cdot 10^{-5} \cdot \sqrt{J} \cdot E_0^{3/2} + 0.735 \cdot 10^{-6} \cdot E_0^2 \right)$$

$$\eta = E_0^m \left[ 0.1904 - 0.2236 \cdot \ln Z + 0.1292 \cdot (\ln Z)^2 - 0.0149 \cdot (\ln Z)^3 \right]$$

$$U_0 = \frac{E_0}{E}, \quad J = 0.0135 \cdot Z, \quad m = 0.1382 - \frac{0.9211}{\sqrt{Z}}$$

($\rho$, density in g cm$^{-3}$, $z$, linear dimension of depth in cm, $\rho x$, depth expressed in mass per unit area (g cm$^{-2}$), $A$, relative atomic weight, $E_0$, electron energy eV in keV corresponding to an acceleration voltage $V$ in kV, $Z$, atomic number of the target material). The probability $dp$ for generation of x-ray photons in depth ($\rho x$) to ($\rho x$)+$d(\rho x)$ is proportional to $d(\rho x)$. With

$$\int_0^{2\rho x} dp = k \cdot \int_0^{2\rho x} d(\rho x) = 1$$

(2)

follows the proportionality factor

$$k = \frac{1}{2\rho x}$$

(3)

and

$$dp = \frac{d(\rho x)}{2\rho x}$$

(4)

This expression is valid for normal electron incidence with regard to the target surface. An incidence angle $\phi$ causes a decrease of the depth $2\rho x$ to $2\rho x \cdot \sin \phi$. Therefore, $dp$ becomes
\[ dp = \frac{d(\varphi)}{2\varphi \sin \varphi} \]  

(5)

Considering the photoelectric absorption of photons of energy \( E \) in the target of element \( j \) a contribution \( d^2N_{E,\text{theor}} \)

\[ d^2N_{E,\text{theor}} = \Omega \cdot i \cdot \sigma_{SR,E} \cdot dE \cdot dp \cdot e^{-\tau_{E,j}/\varphi / \sin \varepsilon} \]  

(6)

(\( \tau_{E,j} \). mass absorption coefficient for photons of energy \( E \) in element \( j \) in cm\(^2\) g\(^{-1}\), \( \varphi \). take-off angle of x-rays, \( \Omega \). solid angle of photon flux from point source to detector in sr, \( i \). x-ray tube current in mA, \( \sigma_{SR,E} \). modified Kramers cross-section\(^{10}\)) is found. After integration from (\( \varphi \)=0 to (\( \varphi \)=2\( \varphi \) \sin \varphi) the pseudomonochromatic contribution \( dN_{E,\text{theor}} \) of photons with energies from \( E \) to \( E + dE \) to continuous x-radiation

\[ dN_{E,\text{theor}} = \Omega \cdot i \cdot \text{const} \cdot Z \cdot \left( \frac{E_0}{E} - 1 \right)^x \cdot \frac{1 - e^{-\tau_{E,j}/2\varphi / \sin \varepsilon}}{\tau_{E,j} \cdot 2\varphi / \sin \varepsilon} \cdot dE \]  

(7)

\[ [dN_{E,\text{theor}}] = s^{-1} \]

is obtained.

For the characteristic countrates the following expression is valid:

\[ N_{jkl} = \Omega \cdot i \cdot \text{Const}_{kl} \cdot \frac{1}{S_{jk}} \cdot R \cdot \omega_{jk} \cdot P_{jkl} \cdot f(\chi_{jkl}) \]  

(8)

\[ [N_{jkl}] = s^{-1} \]

with the absorption term

\[ f(\chi_{jkl}) = \frac{1 - \exp(-\tau \cdot 2\varphi / \sin \varepsilon / \sin \varepsilon)}{\tau \cdot 2\varphi / \sin \varepsilon / \sin \varepsilon} \]  

(9)

(\( j \). gives the element, \( k \). gives the ionized atomic level, \( l \). gives the level from where the \( k \)-vacancy is filled, \( 1/S_{jk} \). stopping power factor, \( P_{jkl} \). transition probability, \( \omega_{jk} \). fluorescence yield, \( R \). backscattering factor, \( \tau \). photoelectric absorption coefficient of characteristic x-rays in the target).

The stopping power factor \( 1/S_{jk} \) is given by
\[
\frac{1}{S_{jk}} = \frac{z_k b_k}{Z} \left( U_0 \ln U_0 + 1 - U_0 \right) \left[ 1 + 16.05 \sqrt{\frac{J}{E_{jk}}} \sqrt{\frac{U_0 \ln U_0 + 2(1 - \sqrt{U_0})}{U_0 \ln U_0 + 1 - U_0}} \right]
\]  

(10)

with

\[z_k = 2, \quad z_L = 8\]
\[b_k = 0.35, \quad b_L = 0.25\]

\[U_0 = \frac{E_0}{E_{jk}}\]
\[J = 0.0135Z\]

and energy \(E_{jk}\) of the absorption edge of the corresponding characteristic radiation, \(z_k\) and \(b_k\) are valid for K-shells and \(z_L\) and \(b_L\) for L-shells. In case of characteristic radiations the theory is based on the absorbed current and thus, measured total currents have to be corrected for backscattering of electrons. A comparably simple expression for the backscattering factor \(R\) is given by Myklebust\(^\text{13}\)

\[R = 1 - 0.0081517 \cdot Z + 3.613 \cdot 10^{-5} \cdot Z^2 + 0.009583 \cdot Z \cdot e^{-U_0} + 0.001141 \cdot E_0\]

(12)

Wiederschwinger\(^\text{4}\) and Ebel et al\(^\text{5}\) found from their evaluations the following numerical values of \(const, x\) and \(Const\)

\[const = 1.36 \cdot 10^9 \text{ sr}^{-1} \text{ mA}^{-1} \text{ keV}^{-1} \text{ s}^{-1} \text{ and } \sigma_{const} = 12\%\]
\[x = 1.0314 - 0.0032 Z + 0.0047 E_0\]

\[Const = 5 \cdot 10^{13} \text{ s}^{-1} \text{sr}^{-1} \text{mA}^{-1} \text{ and } \sigma_{Const} = 19\% \text{ (for K-radiation)}\]

Similar results have been published by Schoßmann et al\(^\text{6}\)

\[const = 1.35 \cdot 10^9 \text{ sr}^{-1} \text{ mA}^{-1} \text{ keV}^{-1} \text{ s}^{-1}\]
\[x = 1.109 - 0.00435 Z + 0.00175 E_0\]

\[Const = 5 \cdot 10^{13} \text{ s}^{-1} \text{sr}^{-1} \text{mA}^{-1} \text{ and } \sigma_{Const} = 19\% \text{ (for K-radiation)}\]

**EXPERIMENTS**

Measurements were performed with an electron probe microanalyzer JEOL JSM-T330, operated at total (cup) current of 60 pA, acceleration voltages of 10, 20 and 30 kV, a time of data acquisition of 400 s, solid angle of x-ray detection of 0.021 sr and fixed angle of 55° between the directions of electron beam and take-off of x-rays. Characteristic data of the x-ray detector were Si(Li)-detector LINK analytical (Model 5508) with 8 µm Be-window, 0.02 µm Au - layer, 0.1 µm inactive Si-layer and active Si thickness of 2 mm. Detector efficiencies have been computed by the method of Fiori et al\(^\text{14}\). The channel width \(\Delta E = 20\text{ eV}\) of the multichannel analyzer defines the step width \(dE\) of the measured spectral responses. All experiments have been performed under fixed geometry between directions of electron beam and x-ray detection. Evaluation of experimental results has been performed by least squares fits of theoretical to experimental responses.
In the first series of experiments the targets have been turned around an axis normal to the plane formed by the electron beam and take-off of x-rays. Thus, the experiments are characterized by \( \varphi + \varepsilon + 55^\circ = 180^\circ \). Results of the evaluation of measured spectra are given in Figs. 2 - 4.

**Fig. 2** Values of \( const \) for white x-radiation of Al, Cu and Mo versus electron incidence angle \( \varphi \).

**Fig. 3** Values of \( x \) for white x-radiation of Al, Cu and Mo versus electron incidence angle \( \varphi \).
**Fig. 4** Values of Const for characteristic x-radiation (Al K, Cu Kα and Mo Kα) versus electron incidence angle $\varphi$.

**Fig. 5** Values of const for white x-radiation (Cu target, acceleration voltage $V = 30$ kV) versus $90^\circ - \varepsilon$. 
In the second series of experiments the targets have been arranged under an angle of 30° between the surface normal and the direction of the electron beam. The targets could be rotated around an axis parallel to the electron beam. Thus, the experiments are characterized by $\phi = 60°$ and $5° < \varepsilon < 65°$. Results of the evaluation of measured spectra are given in Figs. 5 - 7.

**Fig. 6** Values of $x$ for white x-radiation (Cu target, acceleration voltage $V = 30$ kV) versus 90° - $\varepsilon$.

**Fig. 7** Values of $\text{Const}$ for Cu Kα radiation (acceleration voltage $V = 30$ kV) versus 90° - $\varepsilon$. 
From the results of the two series of experiments it becomes evident that the description of electron excited white and characteristic x-ray spectra by Love and Scott’s DDF holds for acceleration voltages of $10 \, \text{kV} < V < 30 \, \text{kV}$, the elemental range from Al to Mo, incidence angles of electrons $30^\circ < \varphi < 90^\circ$ and take-off angles of x-rays $15^\circ < \varepsilon < 90^\circ$.

REFERENCES