QUANTITATIVE TRANSPORT PROPERTIES OF GRANULAR MATERIAL CALCULATED FROM X-RAY $\mu$CT IMAGES

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ABSTRACT

X-ray tomography is an accurate and non-invasive technique that measures the internal 3D structure and composition of materials. To enhance and add predictive power to the collected data, various analysis techniques have been developed. In this paper, transport properties calculated directly from the pore space morphology are presented for 36 porous granular specimens. Results for permeability, conductivity, NMR response and drainage capillary pressure are shown to be in excellent agreement with experimental measurements over a wide range of porosities. The results clearly demonstrate the potential of computing a range of physical properties directly on 3D digitized images generated from X-ray $\mu$CT.

INTRODUCTION

A long standing and crucial problem in the study of porous media is to relate the transport properties of a material to its microstructural properties. For example, permeability (the hydraulic conductance of a saturated fluid within a porous medium) has been correlated to porosity (volume fraction of pore space) at a wide range of length scales (e.g., surface-to-volume, pore size from mercury drainage, pore size from NMR relaxation, grain size). Direct testing of these correlations has been limited to a periodic array of spheres [1], model random sphere packs [2, 3] and stochastic reconstructions of porous materials [4]. In this paper we test these correlations directly on rock and soil microstructures generated from 3D $\mu$CT images.

In previous work we have described 3D $\mu$CT [5, 6] studies of a number of specimens, including sandstone core material (2mm to 1cm in diameter) from a range of oil reservoirs and groundwater stores. The cores included homogeneous sandstones, unconsolidated sands and consolidated reservoir sands. The specimens exhibit a broad range of pore and grain sizes, porosity, permeability and tortuosity. After phase separation [7], results can be computed directly from the digitized specimen for a range of geometrical and morphological parameters as well as mechanical and transport properties; these include pore size [8, 9], hydraulic radii, pore and throat sizes [10], NMR relaxation spectra [11] grain size, fabric and texture [12], conductivity [13], permeability [14], mechanical properties [15] and drainage capillary pressure [16]. In this paper, over 4000 independent measures of microstructural transport properties computed from 36 specimens across a range of rock type are reported. This allows for an extensive test of common empirical correlations between fluid permeability and other geometrical/petrophysical parameters.
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METHODOLOGY

A high-resolution and large-field X-ray μCT facility has been used [18, 5, 6] to image all the specimens; most images are acquired at 2048³ voxels. The resolution chosen is dependent on the pore size of the material. For most sandstones studied we observe grains of 100-300 μm and 4-10 μm resolution is sufficient [8, 17]. The limestone specimen is imaged at 5 μm resolution over a 1cm field of view. The specimen with the smallest pores, a carbonate specimen [9], is imaged at 1.3 μm voxel size over a 2.5 mm field of view. A total of 36 specimens were imaged and classified as 5 rock types. Fig. 1 defines the rock types and shows cross-sectional examples of each rock type.

Figure 1: Representative slice of each rock type studied: (a) Homogeneous sand: Fontainebleau sandstone [19] and Berea sand, (b) Unconsolidated sand: two idealized grain packs, two clean soil specimens from a groundwater reservoir, two silty soil specimens and four poorly consolidated reservoir cores from a single reservoir, (c) Consolidated sand: 23 reservoir sandstone cores from different reservoirs, (d) Poorly sorted reservoir sand, (e) Limestone: a very high porosity/permeability quarried limestone core and (f) Carbonate: a vuggy reservoir carbonate core exhibiting a broad range of pore sizes. Some movies are available at URL: xct.anu.edu.au

Permeability Correlations

Fluid permeability depends on the size of the pore throats. The following 4 correlations are based on empirical measurements and rely on different measures of pore throat size. The following parameters are defined: The fluid permeability ($k$), the porosity ($\phi$), the volume ($V_p$) and surface ($S$) of the pore space, a critical pore diameter ($l_c$), the logarithmic mean of the NMR relaxation time ($T_{2m}$) and the tortuosity ($\tau$), which is calculated by conductivity ($\sigma$) computations of water saturated pore space, such that $\phi/\tau$ is defined as $\sigma_{saturated\ rock}/\sigma_{water}$. 
Hydraulic Radius Theory

Better known as the Kozeny-Carman Relation, the pore length scale is provided by the ratio \( V_p/S \).

\[
k = c_H \frac{\phi(V_p/S)^2}{2\tau}
\]

Critical Pore Diameter

Better known as the Katz-Thompson Relation [20], they argued that the effective permeability of a rock is controlled by \( l_c \), a critical pore diameter corresponding to the diameter of the smallest pore of the set of largest pores that percolate through the rock. The constant \( c_{kt} \) depends on the distribution of pore sizes. The value of \( c_{kt} \) derived in [20] was \( c_{kt} \approx 1/226 \). More recent work suggests that the correct value should be larger by a factor of 2–11 [21, 22, 23]. A feature of this method is that \( l_c \) can be directly measured from mercury intrusion experiments.

NMR Permeability

The connection between NMR relaxation measurements and permeability stems from the strong effect that the rock surface has on promoting magnetic relaxation. These two permeability correlations [21, 24] are based on \( T_{2lm} \), which is assumed to be related to an average \( V_p/S \) or pore size.

\[
k = a_1 \phi^2 T_{2lm}^2
\]

\[
k = a_2 \phi T_{2lm}^2
\]

Numerical Computation of Petrophysical Properties

The 3D numerical methods used to calculate pore morphology [16, 8, 10], grain fabric and texture [12] and various petrophysical properties [6, 13, 14, 11] directly on the digitized specimen have been published elsewhere. Previous research has shown [8, 9, 13, 14, 17] meaningful predictions are obtained if the specimen size is 8–10 times larger than a primary statistical length scale (either grain or pore size). For the 36 specimens analysed, approximately 60³ grains are imaged. For each specimen, computations were performed on more than 100 sub-volumes. Consequently, over 4000 independent measures of \( k, \phi, l_c, \tau, V_p/S, \) and \( T_{2lm} \) were obtained. This represents a significant multiplier on the quantity of data due to the numerical analysis of \( \mu \)CT images. An illustration of the results of a number of the measurements are given in Fig. 2(a-c). Examples of the calculations and the match to experimental data are summarised in Fig. 2(d-f).

RESULTS

From the 4000 independent measures, the best fit values of the prefactors \( c_{kt}, c_H, a_1 \) and \( a_2 \) from Eqns. 1-4 are determined for each of the five rock types and for all rocks combined. These values, plus the quality of fits, are summarized in Table 1. The best fits for all rock types are summarized in Fig. 3.

The least variation is for \( c_{kt} \), which implies \( l_c \) accurately predicts fluid permeability, for all specimens analysed. A second advantage of Eqn. 2 is that \( l_c \) can be calculated in any direction. This allows for anisotropy in permeability to be observed, even in extreme cases, such as thinly bedded sands [25], where there is variation in permeability of more than one order of magnitude. Note that the value of \( c_{kt} \) is very similar to that derived for a simple bundle of capillary tubes and from critical path analysis of pore networks of low coordination [26] and is nearly one order of magnitude larger than the prediction of [20].
Figure 2: (a) Skeleton of a 300³ voxel sub-volume. (b) Grain pack after grain separation with colours labelling the distinct grains [12]. (c) A cross-section of a reservoir core during drainage at an intermediate saturation. Grains are black, pores are grey and the non-wetting phase within the pores is white. (d) Prediction for the $k$ vs. $\phi$ relationship of 4 small (5 mm) plugs from a single well of a gas reservoir, compared to laboratory data obtained on 60 cores [8]. (e) Equivalent pore radius from digital analysis on a plug and MICP data on the same and a sister plug (unpublished data). (f) Comparison of grain size distributions for an unconsolidated sand obtained digitally to one obtained by laser particle sizing on a sister plug [12].

<table>
<thead>
<tr>
<th>Rock Type</th>
<th>$c_{kt}$</th>
<th>$c_H$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$S^2(c_{kt})$</th>
<th>$S^2(c_H)$</th>
<th>$S^2(a_1)$</th>
<th>$S^2(a_2)$</th>
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<td>.0033</td>
<td>.067</td>
<td>.26</td>
<td>.35</td>
<td>.27</td>
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<tr>
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<td>.016</td>
<td>.0066</td>
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<td>.035</td>
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<td>.0075</td>
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<td>Consolidated</td>
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<tr>
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<td>.054</td>
<td>.089</td>
<td>.156</td>
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</table>

Table 1. Prefactors and errors for the permeability correlations in Eqns. 1-4 across the five rock types. The best fits to the data were calculated using linear regression, with the mean residual error given by:

$$S^2 = \frac{1}{N} \sum_{i=1}^{N} [\log(<\text{fit}>_i) - \log(<\text{data}>_i)]^2.$$ 

The other 3 correlations, Eqns. 1,3,4, are based on a length scale associated with the average pore size and give poorer correlations. In Fig. 3, one observes that $c_H$ varies slightly but with a slight mismatch for smaller values of permeability. Also, the variation of $a_1$ is much larger compared to the more robust values of $a_2$. Considering the variation in rock types and permeabilities, the fits are quite good. This suggests that throat sizes are correlated to pore sizes in these rocks. While this is known for sands, it is perhaps a surprising result for carbonates. To further test this result, more carbonate cores must be analysed. One also observes a similar match between $c_H$ and $a_2$, indicating that $V_p/S$ correlates strongly to $T_{21m}$. This shows the importance of accurately estimating $\tau$ in permeability correlations.
The results clearly demonstrate the potential of computing a range of petrophysical properties directly on 3D digitized images generated from X-ray μCT. Over 4000 independent measures from 36 cores for a range of rock and soil specimens were used to directly test widely used empirical correlations between fluid permeability and other petrophysical parameters. All correlations, Eqns. 1-4, perform well, suggesting that throat sizes are correlated to pore sizes in these cores. However, Eqn. 2 is the most accurate, reflecting that \( l_c \) is a good measure of throat radii. Comparison of equations 3 and 4 illustrates importance of an accurate measurement of tortuosity when predicting permeability. Further work is required to extend the study to a wider range of carbonate specimens. Extension to studies of correlations for relative permeability and elastic properties are also underway.
ACKNOWLEDGEMENTS

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REFERENCES