DUAL-AXIS HOLE-DRILLING ESPI RESIDUAL STRESS MEASUREMENTS

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ABSTRACT

A dual-axis ESPI hole-drilling residual stress measurement method is described. It has the advantage of evaluating all the in-plane normal stress components with similar response to measurement errors, significantly lower than with single-axis measurements. Experimental tests were done on a stressed specimen, and the results supported theoretical expectations.

INTRODUCTION

Residual stresses play an important role in the strength and dimensional stability of engineering components. The stresses can be either beneficial or harmful, depending on their distribution [1,2]. The dimensional changes that occur when stressed material is removed forms the basis of the “destructive” methods for residual stress measurement. Of these methods, the hole-drilling method [3,4,5] is the most widely used. The established procedure, standardized as ASTM Standard Test Method E837 [3] uses strain gages to measure the dimensional changes caused by the drilling of a small hole. The residual stresses are then evaluated from the measured dimensional changes. This technique is effective, but is limited by the minimal amount of data that it provides: three strain measurements to identify three in-plane stress components.

In recent years, Electronic Speckle Pattern Interferometry (ESPI) has been introduced as an alternative to strain gauges for hole-drilling measurements [6,7,8,9]. ESPI is a full-field optical technique that gives very sensitive (nanometer range) measurements of surface displacements [10,11], in this case the displacements around a hole drilled in a material containing residual stresses. The full-field character of the technique provides measurements of the displacements at all the pixels imaged around the drilled hole. These can total several hundred thousand or millions of pixels. These numerous pixels provide excellent opportunities for data averaging and consistency checking [12]. However, a typical limitation is that surface displacement components are measured in a specific “sensitivity direction”. The measured displacements are mostly controlled by the stress components that are parallel to the sensitivity direction. For example, a sensitivity vector with a projection in the x-direction will measure surface displacements that are mostly created by the x-stress. The y-stress has much less influence, since the surface displacements it generates are multiplied by Poisson’s ratio, about one third the size. In consequence, the y-stresses are evaluated much less accurately.

In this paper, a novel technique is presented where hole-drilling ESPI measurements are made in two directions, one to favor the x-stress and the second to have increased y-stress sensitivity. In this way, stress evaluation accuracy is better balanced. The doubling of available data increases data averaging and enhances the accuracy of all evaluated stresses.
ESPI HOLE-DRILLING

Figure 1 shows an exaggerated cross-sectional view of the displacements around a drilled hole in a stressed material. The displacements are primarily in-plane, with some out-of-plane displacements due to the Poisson effect. The arrow shows the sensitivity direction of a typical ESPI measurement.

Figure 2 schematically shows a typical ESPI setup [8] for surface displacement measurements. The light from a coherent laser source is split into two parts using a beamsplitter. One part of the beam illuminates the object, which is imaged by a CCD camera. The second part of the laser light passes through an optical fiber directly to the CCD. The two parts of the laser light interfere on the CCD surface to form a speckle pattern. The phase at each pixel of the CCD is determined by taking images at four phase angle steps, controlled by a piezo stepper and mirror [10,11]. Deformation of the object surface caused by hole drilling changes the lengths of the illumination and object beam paths, and hence the measured phases. Subtraction of the pixel phases measured before and after hole drilling gives the surface displacements in the sensitivity direction, from which the in-plane residual stresses can be calculated. The optical arrangement is shown in Figure 2, with the sensitivity direction defined as the bisector of the illumination and object beam directions.
Figure 3 shows a typical hole-drilling ESPI fringe pattern from a setup of the type shown in Figure 2. The unsymmetrical fringe shapes are caused by the inclination of the sensitivity vector in the positive x-direction. The bright fringes have the characteristic of contour lines, indicating half-wavelength increments in the component of the measured surface displacements in the sensitivity direction. The concentric circles enclose the area chosen for data usage. The pixels adjacent to the hole, within the inner dashed circle, are excluded because of de-correlation between the pre- and post-hole images caused by mechanical damage from the exit of the chips during hole drilling. The pixels beyond the outer circle are excluded because they are remote from the hole and have low data content. The 100,000+ pixels within the concentric circles contain the most useful data and are selected for the residual stress calculations.

RESIDUAL STRESS CALCULATION

The ESPI fringe pattern shown in Figure 3 contains $N = 100,000+$ pixels, depending on the resolution of the CCD camera used. From these data, the three in-plane stresses $\tau_{xy}$, $\sigma_x$, and $\sigma_y$ need to be evaluated. The challenge here is to extract the benefit of the abundant data available to achieve accurate and stable results, and to do so in a numerically efficient way. A least-squares approach is taken here [13].

The ESPI fringe pattern shown in Figure 3 is computed using the two-argument arctan function from a set of four phase-stepped CCD images. This calculation introduces a computational artifact because the arctan function gives phase results that lie in the range $-\pi$ to $+\pi$, with modulo $2\pi$ rollover for phases outside that range. The first, and very important, computational step is to “unwrap” the phase to remove the modulo $2\pi$ rollovers and represent the phase distribution as a continuous function [14]. This unwrapping step is of key importance because it makes the subsequent calculations linear and much faster and more stable.

The relationship between the unwrapped phase map and the in-plane residual stresses can be written in matrix form as

$$\mathbf{G} \mathbf{s} = \mathbf{d}$$

where $\mathbf{d}$ is the vector containing the $N$ unwrapped phase data. Solution vector $\mathbf{s}$ contains the three in-plane stresses and three rigid-body displacement components

$$\mathbf{s} = \begin{bmatrix} \tau_{xy} & \sigma_x & \sigma_y & w_o & w_x & w_y \end{bmatrix}^T$$
where \( w_o \) represents a uniform displacement in the sensitivity direction, and \( w_x \) and \( w_y \) represent rotations around the x and y axes. These computed displacements may represent actual rigid-body displacements of the specimen or may be artifacts caused by thermal fluctuations in the air within the laser light path.

Matrix \( G \) has \( N \) rows and 6 columns. It represents the relationship between the phase change at each pixel and the in-plane stress and rigid-body motions, and is computed by finite element modeling of the hole cross-section shown in Figure 1 [15]. Equation (1) is over-determined, and can be solved using the least-squares method, giving

\[
G^T G s = G^T d
\]

**DUAL-AXIS METHOD**

Equation 3 gives solutions for the in-plane stresses \( \tau_{xy} \), \( \sigma_x \) and \( \sigma_y \). The associated rigid-body motions \( w_o \), \( w_x \) and \( w_y \) are simultaneously evaluated, although their specific values are usually not of interest. The optical arrangement in Figure 2 gives a sensitivity vector with a significant component in the x-direction. In Figure 2, the x-direction is to the right, parallel to the specimen surface, the z-direction is perpendicular to the specimen surface, and the y-direction is comes out of the page, parallel to the specimen surface. A typical arrangement is to place the camera in the z-direction, perpendicular to the specimen, and the illumination beam about 45° away in the x-z plane. Since both the illumination and object beams are in the x-z plane, the sensitivity vector is also in that plane, and has no component in the y-direction. Thus, the responses to stresses in the x- and y- directions are substantially different. Figure 4 shows the theoretical fringe patterns for hole drilling in a material with stresses \( \tau_{xy}, \sigma_x \) and \( \sigma_y \) each set at 100 MPa. In these patterns, the x-direction is to the right, and the y-direction is upwards. Figure 4(a) for \( \tau_{xy} \) shows four fringes, Figure 4(b) for \( \sigma_x \) shows three, while Figure 4(c) for \( \sigma_y \) shows just over one. Even though \( \sigma_x \) and \( \sigma_y \) are the same, the setup of the instrument provides better sensitivity in the x-direction.

![Figure 4](image-url)

**Figure 4.** Computed ESPI fringe patterns for in-plane stresses.

(a) \( \tau_{xy} = 100 \) MPa.  
(b) \( \sigma_x = 100 \) MPa.  
(c) \( \sigma_y = 100 \) MPa.
A response to the imbalance in sensitivity to the stresses \( \sigma_x \) and \( \sigma_y \) is to do ESPI measurements in two directions, one favoring \( \sigma_x \) evaluation and one favoring \( \sigma_y \) evaluation. This doubles the measurement and computational effort, but it does not require any substantial conceptual change. Equations 1 and 3 still apply, but with matrix \( \mathbf{G} \) containing twice as many rows, representing the double set of ESPI measurements. Solution vector \( \mathbf{s} \) in Equation 2 also expands to contain separate rigid-body motions in the two directions

\[
\mathbf{s} = [\tau_{xy} \ \sigma_x \ \sigma_y \ \omega_{x1} \ \omega_{y1} \ \omega_{x2} \ \omega_{y2} ]^T \quad (4)
\]

The rigid-body motions in the two measurement directions appear as independent items because their character as artifacts may cause them to differ, even though physically they may be expected to be the same. Vector \( \mathbf{s} \) has nine components, hence the expanded matrix \( \mathbf{G} \) has 9 columns. The lower halves of columns 7-9 duplicate the upper halves of columns 4-6. The remaining halves contain zeroes.

**ERROR SENSITIVITY ANALYSIS**

In common with all other experimental measurements, ESPI data contain noise. This noise gives the fringe pattern in Figure 3 its characteristic grainy appearance. In contrast, the theoretical fringe patterns in Figure 4 have smooth shapes.

Measurement noise creates consequent errors in the computed in-plane stresses. To simplify the analysis, it is assumed that all measurement errors are random and are normally distributed with zero mean. Equation 3 shows that there is a linear relationship between the phase data and the computed stresses. Thus, using the principle of superposition, the error in a calculated stress value is the sum of the stress errors caused by the phase error at each pixel considered one at a time

\[
\text{Error}(\sigma) = \sum_i c_i \text{Error}(\phi)_i \quad (5)
\]

where \( \text{Error}(\phi)_i \) is the phase error at pixel “i” and \( c_i \) is the response factor that defines the stress corresponding to a unit phase value [16]. In the present case, where the phase errors are assumed to be random and normally distributed, the standard deviation of the resulting stress computation error depends on the standard deviation of the phase errors

\[
\text{StdDev}(\sigma) = \sqrt{\sum_i (c_i \text{StdDev}(\phi)_i)^2} \quad (6)
\]

For the case where the standard deviation of the measured phase \( \text{StdDev}(\phi) \) is the same at every pixel, Equation (6) becomes

\[
\text{StdDev}(\sigma) = \sqrt{\sum_i {c_i^2 \text{StdDev}(\phi)}} \quad (7)
\]

The response factors \( c_i \) can be evaluated using Equation 3 by substituting for the data vector \( \mathbf{d} \) a single pixel “i” with unit value, and all other pixels zero. The results computed in this way correspond to \( c_i \). The calculation is repeated for all pixels, from which the standard deviation of the stresses \( \text{StdDev}(\sigma) \) can be determined using Equation 7. Conceptually, the
standard deviations of the rigid-body motions could be similarly evaluated. However, these quantities are measurement artifacts, and so their numerical values are not of interest here.

**EXPERIMENTAL MEASUREMENTS**

Figure 5 shows the dual-axis ESPI system used for this study. It consists of two, single-axis ESPI systems (“Prism”, American Stress Technologies, Cheswick, PA) schematically shown in Figure 2. This arrangement was used because it was easy to set up using existing equipment. It is anticipated that future work will be done using a single-camera system with two switchable illumination beams. Table 1 summarizes the geometric arrangement of the two ESPI systems, where the x- and z- axes are defined in Figure 2, and the y-axis extends upwards, out-of-plane from the table.

![Figure 5. Dual-axis ESPI system.](image)

<table>
<thead>
<tr>
<th></th>
<th>ESPI Axis 1</th>
<th>ESPI Axis 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Illumination Beam</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Angle from z-axis to x-axis</td>
<td>54°</td>
<td>15°</td>
</tr>
<tr>
<td>Angle from z-axis to y-axis</td>
<td>0°</td>
<td>34°</td>
</tr>
<tr>
<td><strong>Object Beam</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Angle from z-axis to x-axis</td>
<td>38°</td>
<td>1°</td>
</tr>
<tr>
<td>Angle from z-axis to y-axis</td>
<td>0°</td>
<td>36°</td>
</tr>
<tr>
<td><strong>Sensitivity Direction</strong></td>
<td>[0.72 0.00 0.69]</td>
<td>[0.11 0.58 0.81]</td>
</tr>
</tbody>
</table>

Table 1. Beam geometry and sensitivity directions of the dual-axis ESPI system.

The geometries of the two perpendicular ESPI systems are generally similar but not identical. The mounting hardware used for the second ESPI system did not have sufficient adjustability to allow exact duplication in the y-z plane of the geometry of the first ESPI system in the x-z plane. However, the first ESPI system was mostly in the x-z plane and the second mostly in the y-z plane.

A test specimen with simulated residual stresses was made by bending a uniform aluminum beam 25.4mm x 12.7mm x 254mm (1” x 1/2” x 10”) in a four-point bend fixture, with an inner support span of 127mm and outer support span of 254mm (5” and 10”). The beam was loaded to produce a previously calibrated known stress of 226 MPa (32.7 Ksi) in the area.
between the inner supports. A 1.59mm (\(\frac{1}{16}\)”) diameter hole was drilled to a depth of 0.476mm (0.0187”) using an electric drill rotating at 9000 rpm. Multiple images were taken using each ESPI system, eight sets of images before and eight after hole drilling. Reference [8] describes the practical details of hole-drilling ESPI measurements.

Figure 6 shows the ESPI fringe patterns from a typical pre- and post-hole image pair, one from each of the two ESPI systems that were used. The circular drilled hole appears as an ellipse because it was viewed obliquely by both ESPI systems. The outer two concentric ellipses indicate the range of pixels chosen to provide data for the residual stress calculations.

![Figure 6. Dual-axis ESPI fringe pattern measurements. (a) Axis 1, (b) Axis 2.](image)

For the sets of eight pre-hole ESPI measurements and eight post-hole measurements, a total of 64 possible combinations of pre- and post-hole measurements are available. Residual stress calculations were made from each combination. Table 2 summarizes the means and standard deviations of the stress calculations using as data the 64 combinations of measurements from ESPI Axis 1, the 64 from ESPI Axis 2, and all sets of measurements combined for the dual axis calculations. The table also includes the theoretically expected standard deviations computed using Equation 7. The standard deviation of the phase measurements StdDev(\(\sigma\)) is estimated to be 0.7 radians. This value equals the observed standard deviation of the residual phase pattern remaining after subtraction of the theoretical phase pattern corresponding to the calculated residual stresses.

<table>
<thead>
<tr>
<th>Stress</th>
<th>Average Stress, MPa</th>
<th>Standard Deviation, MPa</th>
<th>Equation 7, MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Axis 1</td>
<td>Axis 2</td>
<td>Dual</td>
</tr>
<tr>
<td>(\tau_{xy})</td>
<td>-4.7</td>
<td>-4.5</td>
<td>-4.6</td>
</tr>
<tr>
<td>(\sigma_x)</td>
<td>231.6</td>
<td>229.8</td>
<td>231.1</td>
</tr>
<tr>
<td>(\sigma_y)</td>
<td>-2.4</td>
<td>7.9</td>
<td>5.3</td>
</tr>
</tbody>
</table>

Table 2. Means, standard deviations and expected standard deviations of measurement sets.
The average stress values in Table 2 for the individual axis and dual-axis cases are closely similar to the applied stress values. In addition, the standard deviations follow the trends expected from the theoretical fringe patterns in Figure 4. For both axes, the shear stress $\tau_{xy}$ has the largest number of fringes in Figure 4 and the lowest standard deviation. For ESPI axis 1, the x-stress $\sigma_x$ has an intermediate number of fringes and an intermediate standard deviation, followed by the y-stress $\sigma_y$, with the fewest fringes and the largest standard deviation. For ESPI axis 2, the roles of the x- and y- axes are interchanged. As expected, the standard deviations for the dual-axis measurements are consistently lower than the single axis measurements. Figure 7 shows these results graphically. The graphs illustrate the low standard deviation of the shear stress evaluation with both axes, the low standard deviation for the x-stress in axis 1 and for the y-stress in axis 2, and the yet lower standard deviations for all stress components with dual-axis measurements. The theoretically expected values from Equation 7 are closely similar.

![Figure 7. Measured and expected standard deviations of measurement sets. (a) measured values, (b) expected values.](image)

DISCUSSION

The results summarized in Table 2 and Figure 7 show an attractive correspondence between theory and experiment. However, this correspondence is just part of a larger picture. Several measurement error sources are at play in ESPI hole drilling, of which optical noise from repeated measurements is just one. This error source was chosen here as a representative of all error sources because it could be created easily and consistently. Other important factors that can lead to stress evaluation uncertainty include inexact identification of the center and radius of the drilled hole, and the scratching of the specimen surface by the chips exiting the hole, none of which were varied here. A comparison of the ESPI-evaluated and applied stresses in Table 2 suggests that an estimate of an additional order of magnitude uncertainty beyond the values shown in the table could be realistic. This would bring the practical uncertainty to within say 6 MPa, still a very practical range.

Although optical noise is only one small error source among several, the trends that it reveals in the resulting uncertainties of the stress evaluations are representative of all error sources. The connection between low ESPI fringe response to a given stress component and
high stress evaluation uncertainty applies to all error sources, not just for optical noise. Similarly, the use of dual axis measurements to reduce evaluation uncertainty in all stress components also applies generally.

Repeated optical measurements from drilling a single hole were used here to produce the measurement variations used in the error analysis. Future work is planned to develop an integrated dual-axis ESPI system using a single camera and two switchable illumination beams. This arrangement will allow dual-axis measurements to be made with only minimal work beyond that required for a single-axis measurement. When this new system is completed, it will become practical to do a more detailed stress evaluation uncertainty study using multiple, individually drilled holes.

CONCLUSIONS

A dual-axis ESPI hole-drilling residual stress measurement method has been successfully demonstrated. It has the advantage of evaluating all in-plane normal stress components with similar error response. This error response is significantly lower than for single-axis measurements. Experimental tests supported theoretical expectations.

The ESPI arrangement used here consisted of two single-axis ESPI systems, so its use doubled the effort required for one single-axis measurement. Future work is planned to develop a more integrated dual-axis ESPI system using a single camera and two switchable illumination beams. This development will enable practical dual-axis measurements to be made in a similar time to single-axis measurements.

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REFERENCES


